Examples:

(1) $\int \frac{2}{1+\cos 2x}\,dx$

$\int \frac{2}{1+2\cos^2 x-1}\,dx$

$\int \frac{2}{2\cos^2 x}\,dx$

$\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx$

$= \tan x + C$

(2) $\int (\tan x + \cot x)^2 \cdot dx$

$\int (\tan^2 x + \cot^2 x + 2\tan x \cdot \cot x)\,dx$

$\int (\sec^2 x - 1 + \operatorname{cosec}^2 x - 1 + 2 \cdot \tan x \cdot \frac{1}{\tan x})\,dx$

$\int (\sec^2 x + \operatorname{cosec}^2 x - 1 + 1)\,dx$

$= \tan x - \cot x + C$

(3) $\int e^{2\log \sec x}\,dx$

$\int e^{\log \sec^2 x} \cdot dx$

$e^{\log x} = x$

$\int \sec^2 x \cdot dx$

$\tan x + C$

(4) $\int \frac{1+\sin^2 x}{\cos^2 x}\,dx$

$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\right) dx$

$= \int (\sec^2 x + \tan^2 x)\,dx$

$= \int (\sec^2 x + \sec^2 x - 1)\,dx$

$= \int (2\sec^2 x - 1)\,dx$

$= 2 \cdot \tan x - x + C$

(5) $\int \frac{\sec x + \tan x}{\sec x - \tan x}\,dx$

$\int \frac{\sec x + \tan x}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x}\,dx$

$= \int \frac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x}\,dx$

$= \int \frac{\sec^2 x + \tan^2 x + 2\sec x \cdot \tan x}{1 + \tan^2 x - \tan^2 x}\,dx$

$= \int (\sec^2 x + \sec^2 x - 1 + 2\sec x \cdot \tan x)\,dx$

$= \int (2\sec^2 x - 1 + 2\sec x \cdot \tan x)\,dx$

$= 2 \cdot \tan x - x + 2 \cdot \sec x + C$

Home Work

(1) $\int \frac{4 - 5 \sin x}{\cos^2 x} dx$

(2) $\int \frac{x^4 + 2x + 4}{x^2 + 1} dx$

(3) $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x + \cos^2 x} \cdot dx$

(4) $\int \frac{\sec x}{\sec x + \tan x} dx$

(5) $\int \frac{(\sin x + \cos x)}{\sqrt{1 + \sin 2x}} dx$