Type. IV.

(1) $I = \int \underset{I}{e^x} \cdot \underset{II}{\cos x} \cdot dx$

$e^x \int \cos x \cdot dx - \int \left[\frac{d}{dx}(e^x) \cdot \int \cos x \cdot dx\right] dx$

$e^x \cdot \sin x - \int \underset{I}{e^x} \cdot \underset{II}{\sin x} \cdot dx$

$e^x \cdot \sin x - \left[ e^x \cdot \int \sin x \cdot dx - \int \left[\frac{d}{dx}(e^x) \cdot \int \sin x \cdot dx\right] dx \right]$

$e^x \cdot \sin x - \left[ e^x (-\cos x) - \int e^x \cdot (-\cos x) \cdot dx \right]$

$e^x \cdot \sin x + e^x \cdot \cos x - \underbrace{\int e^x \cdot \cos x \cdot dx}_{I}$

$2I = e^x \sin x + e^x \cdot \cos x$

$= e^x (\sin x + \cos x)$

$I = \frac{e^x}{2} (\sin x + \cos x) + C$

Type. V.

$\int \cos^{-1} x \cdot dx$

$\int \underset{I}{\cos^{-1} x} \cdot \underset{II}{1} \cdot dx$

$\cos^{-1} x \cdot \int 1 \cdot dx - \int \left[\frac{d}{dx}(\cos^{-1} x) \cdot \int 1 \cdot dx\right] dx$

$\cos^{-1} x \cdot x - \int \frac{-1}{\sqrt{1-x^2}} \cdot x \cdot dx$

$$= x \cdot \cos^{-1}x + \int \frac{x \cdot dx}{\sqrt{1-x^2}}$$

$$1-x^2 = t$$
$$-2x \cdot dx = dt$$
$$x\, dx = -\frac{dt}{2}$$

$$= x \cdot \cos^{-1}x - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= x\cos^{-1}x - \frac{1}{2}\int t^{-1/2} \cdot dt$$

$$= x \cdot \cos^{-1}x - \frac{1}{2}\left(\frac{t^{-1/2+1}}{-1/2+1}\right)$$

$$= x \cdot \cos^{-1}x - \frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)$$

$$= x\cos^{-1}x - \sqrt{t}$$

$$= x\cos^{-1}x - \sqrt{1-x^2} + C$$

**Work.**

1. $\int e^{-x} \cdot \sin x \cdot dx$
2. $\int e^{x} \sin^2 x \cdot dx$
3. $\int x \log(1+x) \cdot dx$
4. $\int x^3 \cdot \tan^{-1}x \cdot dx$
5. $\int \log(1+x) \cdot dx$