

(2)

PAGE No.

DATE: / /

$$\textcircled{1} \int \frac{dx}{x(x^n+1)}$$

$$= \int \frac{x^{n-1}}{x^n(x^n+1)} dx \quad (\text{multiply by } x^{n-1})$$

$$x^n = t$$

$$n x^{n-1} dx = dt$$

$$= \frac{1}{n} \int \frac{dt}{t(t+1)}$$

$$= \frac{1}{n} \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt$$

$$= \frac{1}{n} \int \frac{1}{t} dt - \frac{1}{n} \int \frac{1}{t+1} dt$$

$$= \frac{1}{n} \log t - \frac{1}{n} \log(t+1) + C$$

$$= \frac{1}{n} \log x^n - \frac{1}{n} \log(x^n+1) + C$$

$$= \frac{1}{n} \log \left(\frac{x^n}{x^n+1} \right) + C$$

$$\textcircled{2} \int \frac{dx}{1+3e^x+2e^{2x}}$$

$$= \int \frac{e^x dx}{e^x+3e^{2x}+2e^{3x}} \quad (\text{multiply by } e^x)$$

$$e^x = t$$

$$e^x dx = dt$$

$$= \int \frac{dt}{t+3t^2+2t^3}$$

$$= \int \frac{dt}{t(1+3t+2t^2)}$$

$$= \int \frac{dt}{t(2t^2+2t+1)}$$

$$= \int \frac{dt}{t[2t(t+1)+1(t+1)]}$$

$$= \int \frac{dt}{t(t+1)(2t+1)}$$

$$\Rightarrow \frac{1}{t(t+1)(2t+1)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{2t+1}$$

$$\Rightarrow 1 = A(t+1)(2t+1) + B \cdot t(2t+1) + C \cdot t(t+1)$$

$$1 = A(2t^2 + t + 2t + 1) + B(2t^2 + t) + C(t^2 + t)$$

$$= t^2(2A + 2B + C) + t(3A + B + C) + A$$

Compare

$$A = 1$$

$$2A + 2B + C = 0$$

$$3A + B + C = 0$$

$$A = 1, B = 1, C = -4$$

$$\Rightarrow \frac{1}{t(t+1)(2t+1)} = \frac{1}{t} + \frac{1}{t+1} - \frac{4}{2t+1}$$

$$= \int \left(\frac{1}{t} + \frac{1}{t+1} - \frac{4}{2t+1} \right) dt$$

$$= \int \frac{1}{t} dt + \int \frac{1}{t+1} dt - 4 \int \frac{1}{2t+1} dt$$

$$= \log t + \log(t+1) - \frac{4}{2} \log(2t+1)$$

$$= \log t + \log(t+1) - 2 \log(2t+1)$$

$$= \log t(t+1) - \log(2t+1)^2$$

$$= \log \frac{t(t+1)}{(2t+1)^2}$$

$$= \log \frac{e^x(e^x+1)}{(2e^x+1)^2} + C$$

② work

$$\textcircled{1} \int \frac{\cos x \, dx}{(1+\sin x)(2+\sin x)}$$

$$\textcircled{2} \int \frac{x}{(x^2-a^2)(x^2-b^2)} dx$$

$$\textcircled{3} \int \frac{dx}{\sin x + 8 \cos x}$$

$$\textcircled{4} \int \frac{x}{x^3+1} dx$$

$$\textcircled{5} \int \frac{\cos \theta}{4 - \sin^2 \theta} d\theta$$