(5)

① $\int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x}\,dx$

$a\cos^2 x + b\sin^2 x = t$

$2a\cos x(-\sin x) + 2b\sin x\cdot\cos x)\,dx = dt$

$2\sin x\cdot\cos x\,(b-a)\,dx = dt$

$\sin 2x\,dx = \frac{dt}{b-a}$

$\int \frac{dt}{(b-a)t}$

$\frac{1}{b-a}\int \frac{dt}{t}$

$\frac{1}{b-a}\cdot\log t + c$

$\frac{1}{b-a}\log(a\cos^2 x + b\sin^2 x) + c$

② $\int \frac{e^x(1+x)}{\cos^2(x\cdot e^x)}\,dx$

$x\cdot e^x = t$

$(x\cdot e^x + e^x\cdot 1)\,dx = dt$

$e^x(1+x)\,dx = dt$

$\int \frac{dt}{\cos^2 t}$

$= \int \sec^2 t\cdot dt$

$= \tan t + C$

$= \tan x e^x + C$

③ $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\,dx$

$= \int \frac{2\sin x\cdot\cos x\cdot dx}{\sin^4 x + \cos^4 x}$

$\cos^4 x$, devide

$= \int \frac{2\tan x\cdot\sec^2 x\,dx}{\tan^4 x + 1}$

$\tan^2 x = t$

$2\tan x\cdot\sec^2 x\cdot dx = dt$

$= \int \frac{dt}{1+t^2}$

$= \tan^{-1} t + C$

$= \tan^{-1}(\tan^2 x) + C$

④ $\int e^{e^{e^x}}\cdot e^{e^x}\cdot e^x\cdot dx$

$e^{e^{e^x}} = t$

$e^{e^{e^x}}\cdot e^{e^x}\cdot e^x\cdot dx = dt$

$\int dt$

$= t + C$

$= e^{e^{e^x}} + C$

(5) $\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$

$1+\sqrt{x} = t^2$

$\sqrt{x} = t^2 - 1$

$x = (t^2-1)^2$

$dx = 2(t^2-1)\cdot 2t\cdot dt$

$\int \frac{4t(t^2-1)\,dt}{t}$

$4\int (t^2-1)\,dt$

$= 4\left[\frac{t^3}{3} - t\right] + C$

$= \frac{4}{3}\left(\sqrt{1+\sqrt{x}}\right)^3 - 4\sqrt{1+\sqrt{x}} + C$

Work.

(1) $\int \frac{5}{\sin^2 x\cdot\cos^3 x}\,dx$

(2) $\int \frac{x\tan^{-1}x^2}{1+x^4}\,dx$

(3) $\int \frac{1}{1+\tan x}\,dx$

(4) $\int \frac{dx}{\sqrt{x}\sqrt{1-x}}$

(5) $\int \sqrt{1+2\tan x(\tan x+\sec x)}\,dx$