Type. VI.

(1) $\int \sin^{-1} \sqrt{\frac{x}{a+x}} \cdot dx$

$x = a\tan^2\theta$

$dx = 2a \cdot \tan\theta \cdot \sec^2\theta \cdot d\theta$

$= \int \sin^{-1} \sqrt{\frac{a\tan^2\theta}{a + a\tan^2\theta}} \; 2a \cdot \tan\theta \cdot \sec^2\theta \cdot d\theta$

$= 2a \int \sin^{-1} \sqrt{\frac{\tan^2\theta}{\sec^2\theta}} \; \tan\theta \cdot \sec^2\theta \cdot d\theta$

$= 2a \int \underline{\sin^{-1}(\sin\theta)} \cdot \tan\theta \cdot \sec^2\theta \cdot d\theta$

$= 2a \int \underset{\mathrm{I}}{\theta} \cdot \underset{\mathrm{II}}{\underline{\tan\theta \cdot \sec^2\theta}} \cdot d\theta$

$= 2a \left[ \theta \cdot \int \tan\theta \cdot \sec^2\theta \cdot d\theta - \int \left[ \frac{d}{d\theta}(\theta) \cdot \int \tan\theta \cdot \sec^2\theta \cdot d\theta \right] d\theta \right]$

$\left( \int \tan\theta \cdot \sec^2\theta \cdot d\theta \right.$

$\tan\theta = t$

$\sec^2\theta \cdot d\theta = dt$

$\int t \cdot dt = \frac{t^2}{2}$

$\left. = \frac{\tan^2\theta}{2} \right)$

$= 2a \left[ \theta \cdot \frac{\tan^2\theta}{2} - \int 1 \cdot \frac{\tan^2\theta}{2} \, d\theta \right]$

$= a \cdot \theta \cdot \tan^2\theta - a \int \tan^2\theta \cdot d\theta$

$= a \cdot \theta \cdot \tan^2\theta - a \cdot \int (\underline{\sec^2\theta} - 1) \, d\theta$

$= a \cdot \theta \cdot \tan^2\theta - a \cdot \tan\theta + a \cdot \theta + C$

$$= a \cdot \tan^{-1}\sqrt{\frac{x}{a}} \cdot \frac{x}{a} - a \cdot \sqrt{\frac{x}{a}} + a \cdot \tan^{-1}\sqrt{\frac{x}{a}} + c$$

$$= x \cdot \tan^{-1}\sqrt{x/a} + a \cdot \tan^{-1}\sqrt{x/a} - \sqrt{ax} + c$$

$$= (x+a)\tan^{-1}\sqrt{x/a} - \sqrt{ax} + C$$

Type. VII

(2) $\int e^x(\log\sin x + \cot x)\,dx$

$$= \int e^x \log\sin x \cdot dx + \int \underset{I}{e^x} \cdot \underset{II}{\cot x} \cdot dx$$

$$= \int e^x \cdot \log\sin x \cdot dx + e^x \cdot \int \cot x \cdot dx - \int\left[\frac{d}{dx}(e^x) \cdot \int \cot x \cdot dx\right]dx$$

$$= \cancel{\int e^x \cdot \log\sin x \cdot dx} + e^x \cdot \log\sin x - \cancel{\int e^x \cdot \log\sin x\, dx}$$

$$= e^x \cdot \log\sin x + C$$

Work.

(1) $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

(2) $\int \frac{x^2\tan^{-1}x}{1+x^2}\,dx$

(3) $\int \frac{\log(\log x)}{x}\,dx$

(4) $\int \tan^{-1}\sqrt{\frac{a-x}{a+x}}\,dx$

(5) $\int \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\}dx$