

Integration

Trigonometry

माना $\sin x = y$
 $\Rightarrow \cos x \cdot dx = dy$
 $\Rightarrow \int \cos x \cdot dx = \int dy = y + C = \sin x + C$

Ex 1.

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\int \cos x \cdot dx = \sin x$$

$$\frac{d}{dx}(\sin x + 2) = \cos x$$

$$\int \cos x \cdot dx = \sin x + 2$$

माना $\cos x = y$
 $\Rightarrow -\sin x \cdot dx = dy$
 $\Rightarrow \int -\sin x \cdot dx = \int dy = y + C = \cos x + C$

$$\Rightarrow \int \sin x \cdot dx = -\cos x + C$$

$$\Rightarrow \int \cos x \cdot dx = \sin x$$

$$\Rightarrow \int \sec^2 x \cdot dx = \tan x$$

$$\Rightarrow \int \sec x \cdot dx = \ln |\sec x + \tan x|$$

$$\Rightarrow \int \sec x \cdot \tan x \cdot dx = \sec x$$

$$\Rightarrow \int \csc x \cdot dx = -\ln |\csc x + \cot x|$$

$$\Rightarrow \int \frac{1}{1-x^2} dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$$

$$\downarrow$$

$$-\cot^{-1} x$$

$$\Rightarrow \int \frac{1}{1+x^2} dx = \tan^{-1} x$$

$$\downarrow$$

$$-\cot^{-1} x$$

$$\Rightarrow \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x$$

$$\downarrow$$

$$-\log_e |x|$$

$$(1) \int (x^3 + 3x^2 + 3x + 1) dx$$

$$= \frac{x^4}{4} + 3 \cdot \frac{x^3}{3} + 3 \cdot \frac{x^2}{2} + x + C$$

$$\Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\Rightarrow \int \frac{1}{x} dx = \log x + C$$

$$\Rightarrow \int e^x \cdot dx = e^x$$

$$\Rightarrow \int a^x \cdot dx = \frac{a^x}{\log a}$$

$$(2) \int \frac{(a^x + b^x)^2}{a^x \cdot b^x} dx$$

$$= \int \frac{a^{2x} + b^{2x} + 2 \cdot a^x \cdot b^x}{a^x \cdot b^x} dx$$

$$= \int \left(\frac{a^x}{b^x} + \frac{b^x}{a^x} + 2 \right) dx$$

$$\int \left[\left(\frac{a}{b} \right)^x + \left(\frac{b}{a} \right)^x + 2 \right] dx$$

$$= \frac{(a/b)^x}{\log_e a/b} + \frac{(b/a)^x}{\log_e b/a} + 2x + C$$

$$= (a/b)^x \cdot \log_e b/a + (b/a)^x \cdot \log_e a/b + 2x + C$$

$$+ C \quad (3) \int \frac{1}{1 + \sin 2x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x}{\sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x} dx = \int \frac{\sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x}{(\sin x + \cos x)^2} dx$$

$$= \int (\sin x + \cos x)^2 dx$$

$$= \int (\sin x + \cos x) dx$$

$$= -\cos x + \sin x + C$$

$$(4) \int \frac{x+1}{x+2} dx$$

$$\int \left(1 - \frac{1}{x+2} \right) dx$$

$$= x - \log(x+2) + C$$

$$(5) \int \frac{1}{1 + \sin x} dx$$

$$\int \frac{1}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x} dx$$

$$\int \frac{1 - \sin x}{1 - \sin^2 x} dx$$

$$= \int \frac{1 - \sin x}{\cos^2 x} dx$$

$$= \int (\sec^2 x - \sec x \cdot \tan x) dx$$

$$= \tan x - \sec x + C$$

HOME WORK

$$(1) \int \frac{(1-x^2)^3}{x^2} dx$$

$$(2) \int (x+1)^2 dx$$

$$(3) \int \frac{(x+a)^3}{\sqrt{x}} dx$$

$$(4) \int \frac{1}{\sin x \cdot \tan x} dx$$

$$(5) \int \frac{(x+1)(x-2)}{(x+4)} dx$$

dx

HOME WORK

$$\textcircled{1} \int \frac{(1-x^2)^3}{x^2} dx$$

$$\textcircled{2} \int (x+1)^2 \sqrt{x} dx$$

$$\textcircled{3} \int \frac{(x+a)^3}{\sqrt{x}} dx$$

$$\textcircled{4} \int \frac{1}{\sin x \cos x} dx$$

$$\textcircled{5} \int \frac{(x-1)(x-2)}{(x+4)} dx$$