Applied Physics

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EE I (morning), C.S.I., M.Cad I

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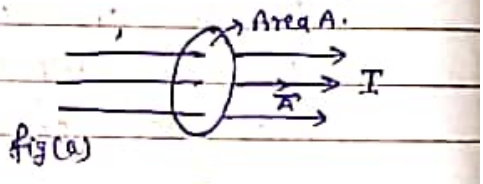
Q: Define the terms current density, conductance and conductivity. Write their SI units. Express Ohm's Law in vector form.

Ans. Current density:

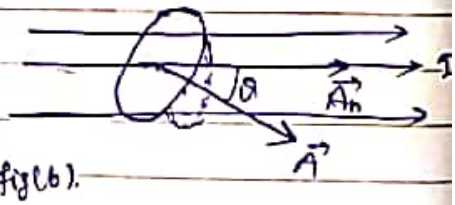
The current density at any point inside a conductor is defined as the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point.

It is a vector quantity having the same direction as that of the motion of the positive charge. It is a characteristic property of any point inside the conductor and is denoted by $\vec{J}$.

As shown in fig(a), if a current I is flowing uniformly and normally through an area of cross-section A of a conductor, then the magnitude of current density at any point of this cross-section will be-



fig(a)



fig(b).

$$j = \frac{q/t}{A} = \frac{I}{A}$$

If the area A is not perpendicular to the direction of current and normal to this area makes angle θ with the direction of current as shown in fig(b), then the component of A normal to the direction of current flow will be -

$$A_n = A\cos\theta$$

$\therefore$ current density

$$J = \frac{I}{A_n} = \frac{I}{A\cos\theta}$$

$$\boxed{I = JA\cos\theta = \vec{J}\cdot\vec{A}}$$

This equation again shows that electric current, being scalar product of two vectors, is a scalar quantity.

The SI unit of current density is ampere per square meter (Am^{-2}) and its dimension are $[AL^{-2}]$.

* The current I through a particular surface S in a conductor is the flux of $\vec{J}$ through that surface and is given by the surface integral.

$$I = \int_S \vec{J}\cdot d\vec{s}$$

where $d\vec{s}$ is a small element of the given surface area.

Conductance: The conductance of a conductor is the ease with which electric charges flow through it. It is equal to the reciprocal of its resistance and is denoted by C.

Thus,

$$\text{Conductance} = \frac{1}{\text{Resistance}}$$

$$\boxed{C = \frac{1}{R}}$$

The SI unit of conductance is ohm^{-1} or mho or siemens (S).

Conductivity: The reciprocal of the resistivity of a material is called its conductivity and is denoted by σ.

Thus,

$$\text{conductivity} = \frac{1}{\text{Resistivity}}$$

$$\boxed{\sigma = \frac{1}{\rho}}$$

The SI unit of conductivity is $ohm^{-1}\,m^{-1}$ or $mho\,m^{-1}$ or $S\,m^{-1}$.

Vector form of Ohm's Law:

If E is the magnitude of electric field in a conductor of length l, then the potential difference across its ends is

$$V = El$$

Also from ohm's law, we have -

$$V = IR = I\frac{\rho l}{A}$$

$$El = \frac{I}{A}\rho l$$

$$\boxed{E = J\rho}$$

As the direction of current density J is same as that of electric field $\vec{E}$,

we can write the above equation as-

$$\vec{E} = \rho \vec{J}$$

$$\boxed{\vec{J} = \sigma \vec{E}}$$

The above equation is the vector form of ohm's law. It is equivalent to the scalar form $V = RI$.