

①

PAGE NO:

DATE: / /

Integration by Partial Fraction:

Type-I.

$$\frac{x-1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

Type-II.

$$\frac{1}{(1-x)(1+x)^2} = \frac{A}{1-x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

Type-III

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

Type-IV

$$\int \frac{\sin x \cdot \cos x}{\cos^2 x - 5 \cos x + 4} dx, \quad (\cos x = t)$$

①

$$\int \frac{x^2 \cdot dx}{(x-1)(3x-1)(3x-2)} = \int \frac{A}{x-1} dx + \int \frac{B}{3x-1} dx + \int \frac{C}{3x-2} dx$$

$$\Rightarrow \frac{x^2}{(x-1)(3x-1)(3x-2)} = \frac{A}{x-1} + \frac{B}{3x-1} + \frac{C}{3x-2} \quad \text{--- (1)}$$

$$x^2 = A(3x-1)(3x-2) + B(x-1)(3x-2) + C(x-1)(3x-1)$$

$$x=1, \quad A = \frac{1}{2}$$

$$x = \frac{1}{3}, \quad B = \frac{1}{6}$$

$$x = \frac{2}{3}, \quad C = -\frac{4}{3}$$

$$\frac{1}{2} \int \frac{dx}{x-1} + \frac{1}{6} \int \frac{dx}{3x-1} - \frac{4}{3} \int \frac{dx}{3x-2}$$

$$\frac{1}{2} \log(x-1) + \frac{1}{6} \log(3x-1) - \frac{4}{3} \log(3x-2) + C$$

$$\frac{1}{2} \log(x-1) + \frac{1}{18} \log(3x-1) - \frac{4}{9} \log(3x-2) + C$$

②

$$\int \frac{x-1}{(x-3)(x-2)^2} dx = \int \frac{A}{x-3} dx + \int \frac{B}{x-2} dx + \int \frac{C}{(x-2)^2} dx$$

$$\Rightarrow \frac{x-1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \quad \text{--- (1)}$$

$$x-1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

$$= A(x^2 + 4 - 4x) + B(x^2 - 2x - 3x + 6) + C(x-3)$$

$$= x^2(A+B) + x(-4A-5B+C) + 4A+6B-3C$$

Compare

$$\begin{aligned} A+B &= 0 \\ -4A-5B+C &= 1 \\ 4A+6B-3C &= -1 \end{aligned}$$

$$\begin{aligned} A &= 2 \\ B &= -2 \\ C &= -1 \end{aligned}$$

$$= \int \frac{2}{x-3} dx - \int \frac{2}{x-2} dx - \int \frac{1}{(x-2)^2} dx$$

$$= 2 \log(x-3) - 2 \log(x-2) + \frac{1}{(x-2)} + C$$

$$(3) \int \frac{dx}{(x+1)(x^2+1)} = \int \frac{A}{x+1} dx + \int \frac{Bx+C}{x^2+1} \quad \text{--- (1)}$$

$$\Rightarrow \frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + (Bx+C)(x+1)$$

$$= A(x^2+1) + (Bx^2+Bx+Cx+C)$$

$$= x^2(A+B) + x(B+C) + (C+A)$$

Compare.

$$A+B=0$$

$$B+C=0$$

$$A+C=1$$

$$A = \frac{1}{2}$$

$$B = -\frac{1}{2}$$

$$C = \frac{1}{2}$$

$$\Rightarrow \int \frac{1}{2(x+1)} dx + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} dx$$

$$= \frac{1}{2} \int \frac{1}{x+1} dx - \frac{1}{4} \int \frac{2x}{x^2+1} dx + \frac{1}{2} \int \frac{dx}{x^2+1}$$

$$= \frac{1}{2} \log(x+1) - \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1}x + C$$

Work Ex. (1) $\int \frac{x^2+1}{x(x^2-1)} dx$

(2) $\int \frac{x^3+3}{x^2-3x} dx$

(3) $\int \frac{x^3+2}{(x-2)^3(x-1)} dx$

(4) $\int \frac{x}{x^3+1} dx$

(5) $\int \frac{1}{1+x+x^2+x^3} dx$