Class 21: 4th Lecture

Integration by Parts

$$\int u \cdot v \, dx = u \cdot \int v \cdot dx - \int \left(\frac{d}{dx}(u) \cdot \int v \cdot dx \right) dx$$

Type. I

(1) $\int x \cdot \sin x \cdot \cos x \cdot dx$

$\frac{1}{2} \int x \cdot 2 \cdot \sin x \cdot \cos x \cdot dx$

$\frac{1}{2} \int x \cdot \sin 2x \cdot dx$

$\frac{1}{2} \left[x \cdot \int \sin 2x \cdot dx - \int \left(\frac{d}{dx}(x) \cdot \int \sin 2x \cdot dx \right) dx \right]$

$\frac{1}{2} \left[x \cdot \frac{-\cos 2x}{2} - \int 1 \cdot \frac{(-\cos 2x)}{2} \, dx \right]$

$\frac{1}{2} \left[\frac{-x \cos 2x}{2} + \frac{1}{2} \int \cos 2x \cdot dx \right]$

$\frac{1}{2} \left[-\frac{x \cos 2x}{2} + \frac{1}{2} \cdot \frac{\sin 2x}{2} \right]$

$-\frac{x \cos 2x}{4} + \frac{1}{8} \sin 2x$

Type. II

(2) $\int x^2 \cdot e^{ax} \, dx$

$x^2 \int e^{ax} \cdot dx - \int \left(\frac{d}{dx}(x^2) \cdot \int e^{ax} \cdot dx \right) dx$

$x^2 \cdot \frac{e^{ax}}{a} - \int 2x \cdot \frac{e^{ax}}{a} \, dx$

$\frac{x^2 \cdot e^{ax}}{a} - \frac{2}{a} \int x \cdot e^{ax} \, dx$

I II

$$\frac{x^2 \cdot e^{ax}}{a} - \frac{2}{a}\left[x \cdot \int e^{ax} - \int \left(\frac{d}{dx}(x) \cdot \int e^{ax} \cdot dx\right) dx\right]$$

$$\frac{x^2 \cdot e^{ax}}{a} - \frac{2}{a}\left[x \cdot \frac{e^{ax}}{a} - \int 1 \cdot \frac{e^{ax}}{a} \cdot dx\right]$$

$$\frac{x^2 \cdot e^{ax}}{a} - \frac{2}{a}\left[\frac{x \cdot e^{ax}}{a} - \frac{1}{a} \cdot \frac{e^{ax}}{a}\right]$$

$$\frac{x^2 \cdot e^{ax}}{a} - \frac{2x \cdot e^{ax}}{a^2} + \frac{2}{a^3} \cdot e^{ax}$$

Type-141

(3) $\int x \cdot \tan^2 x \cdot dx$

$\int x \cdot (\sec^2 x - 1)\, dx$

$\int x \cdot \sec^2 x \cdot dx - \int x \cdot dx$

I II

$x \cdot \int \sec^2 x \cdot dx - \int \left(\frac{d}{dx}(x) \cdot \int \sec^2 x\, dx\right) dx - x^2/2$

$x \cdot \tan x - \int 1 \cdot \tan x \cdot dx - x^2/2$

$x \cdot \tan x - \log \sec x - x^2/2 + C$

Work:

(1) $\int x \cdot a^x \cdot dx$

(2) $\int x \cdot e^{3x} \cdot dx$

(3) $\int x^2 \cdot \cos x \cdot dx$

(4) $\int x \cdot \sin^2 x \cdot dx$

(5) $\int x \cdot \sec^2 x \cdot dx$