પ્રતિસ્થાપન દ્વારા સંકલન (1)

Type. I

$\Rightarrow \int (ax \pm b)^n \, dx$

$ax + b = t$

$a \cdot dx = dt$

$\int \frac{t^n}{a} \cdot dt$

$\frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + C$

$\frac{1}{a} \cdot \frac{(ax \pm b)^{n+1}}{n+1} + C$

$\Rightarrow \int (ax \pm b)^n \cdot dx = \frac{(ax \pm b)^{n+1}}{(n+1)a} + C$

$\Rightarrow \int \frac{1}{ax \pm b} \cdot dx = \frac{\log(ax \pm b)}{a} + C$

$\Rightarrow \int \cos(ax \pm b) \cdot dx = \frac{\sin(ax \pm b)}{a} + C$

$\Rightarrow \int e^{ax \pm b} \cdot dx = \frac{e^{(ax \pm b)}}{a} + C$

Ex. $\int (8x+3)^5 \cdot dx$

$8x + 3 = t$

$8 \cdot dx = dt$

$dx = \frac{dt}{8}$

$\int \frac{t^5 \cdot dt}{8}$

$\frac{1}{8} \cdot \frac{t^{5+1}}{5+1} + C$

$\frac{1}{8} \frac{(8x+3)^6}{6} + C$

$\Rightarrow \int (8x+3)^5 \cdot dx = \frac{(8x+3)^6}{48} + C$ **Ex.**

Type. II

$\int \{f(x)\}^2 \cdot f'(x) \cdot dx$

$f(x) = t$

$f'(x) \cdot dx = dt$

$\int t^2 \cdot dt$

$= \frac{t^3}{3} + C$

$= \frac{[f(x)]^3}{3} + C$

Ex. $\int \sec^2 x \cdot \tan^2 x \cdot dx$

$\tan x = t$

$\sec^2 x \cdot dx = dt$

$\int t^2 \cdot dt$

$\frac{t^3}{3} + C$

$\frac{\tan^3 x}{3} + C$

Ex. $\int \frac{f'(x)}{f(x)} \, dx$

$f(x) = t$

$f'(x) \, dx = dt$

$\int \frac{dt}{t} = \log t + C$

$= \log\{f(x)\} + C$

Q: $\int \frac{\cos x}{\sin^3 x} \cdot dx$

$\sin x = t$

$\cos x \, dx = dt$

$\int \frac{dt}{t^3} = \int t^{-3} \cdot dt$

$= \frac{t^{-3+1}}{-3+1} + c$

$= \frac{t^{-2}}{-2} + c$

$= -\frac{1}{2} \frac{1}{t^2} + c$

$= -\frac{1}{2} \frac{1}{\sin^2 x} + c$

$= -\frac{1}{2} \operatorname{cosec}^2 x + c$

$\Rightarrow \int \tan x \cdot dx$

$\int \frac{\sin x}{\cos x} dx$

$\cos x = t$

$-\sin x \cdot dx = dt$

$\int \frac{dt}{t} = -\log t + c$

$= -\log \cos x + c$

$= \log \sec x + c$

$\Rightarrow \int \tan x \cdot dx = \log \sec x + c$

$\Rightarrow \int \cot x \cdot dx = \log \sin x + c$

$\Rightarrow \int \sec x \, dx = \log (\sec x + \tan x)$

$= \log \tan (\pi/4 + x/2) + c$

$\Rightarrow \int \operatorname{cosec} x \cdot dx = \log \tan \frac{x}{2} + c$

$= -\log (\operatorname{cosec} x + \cot x) + c$

$\Rightarrow \int \sin^2 x \cdot dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$

$\Rightarrow \int \cos^2 x \cdot dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$

$\Rightarrow \int \tan^2 x \cdot dx = \tan x - x + c$

$\Rightarrow \int \sin^3 x \cdot dx = \frac{1}{12} \cos 3x - \frac{3}{4} \cos x + c$

$\Rightarrow \int \cos^3 x \cdot dx = \frac{\sin 3x}{12} + \frac{3 \sin x}{4} + c$

$\Rightarrow \int \tan^3 x \cdot dx = \frac{\tan^2 x}{2} - \log \sec x + c$