

(2)

$$\textcircled{1} \int \sin 4x \cdot \cos 3x \cdot dx$$

$$\frac{1}{2} \int 2 \cdot \sin 4x \cdot \cos 3x \cdot dx$$

$$= \frac{1}{2} \int (\sin 7x + \sin x) dx$$

$$= \frac{1}{2} \left[\frac{-\cos 7x}{7} + (-\cos x) \right] + C$$

$$= -\frac{\cos 7x}{14} - \cos x + C$$

$$\textcircled{2} \int \sin^2 x \cdot \cos x \cdot dx$$

$$\sin x = t$$

$$\cos x \cdot dx = dt$$

$$\int t^2 \cdot dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{\sin^3 x}{3} + C$$

$$\textcircled{3} \int \frac{x^5}{1+x^2} dx$$

$$= \int \left(x^3 - x + \frac{x}{1+x^2} \right) dx$$

$$= \int x^3 dx - \int x dx + \frac{1}{2} \int \frac{2 \cdot x}{1+x^2} dx$$

$$= \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2} \log t + C$$

$$= \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2} \log(1+x^2) + C$$

$$\textcircled{4} \int \frac{8 \sec^2 x}{3+4 \tan x} dx$$

$$3+4 \tan x = t$$

$$0+4 \cdot \sec^2 x \cdot dx = dt$$

$$\sec^2 x \cdot dx = \frac{dt}{4}$$

$$= \frac{1}{4} \int \frac{dt}{t}$$

$$= \frac{1}{4} \log t + C$$

$$= \frac{1}{4} \log(3+4 \tan x) + C$$

$$\textcircled{5} \int \frac{1}{e^x + e^{-x}} dx$$

$$\int \frac{1}{e^x + \frac{1}{e^x}} dx$$

$$= \int \frac{e^x}{e^{2x} + 1} dx$$

$$e^x = t$$

$$e^x \cdot dx = dt$$

$$= \int \frac{dt}{t^2 + 1}$$

$$= \tan^{-1} t + C$$

$$= \tan^{-1} e^x + C$$

$$\textcircled{6} \int \frac{a}{b + ce^x} dx$$

$$\int \frac{a}{e^x(b e^{-x} + c)} dx$$

$$\int \frac{a \cdot e^{-x}}{b e^{-x} + c} dx$$

$$b e^{-x} + c = t$$

$$(b \cdot e^{-x}(-1) + 0) dx = dt$$

$$e^{-x} dx = \frac{-dt}{b}$$

$$-\frac{1}{b} \int \frac{a \cdot dt}{t}$$

$$-\frac{a}{b} \int \frac{dt}{t}$$

$$-\frac{a}{b} \log t + C$$

$$-\frac{a}{b} \log(b e^{-x} + c) + C$$

$$\textcircled{7} \int \frac{\sin^2 x}{\sqrt{1-x^2}} dx$$

$$\sin^2 x = t$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\int t \cdot dt = \frac{t^2}{2} + C$$

$$= \frac{(\sin^2 x)^2}{2} + C$$

Work

$$\textcircled{1} \int \left\{ \frac{\log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} \right\} dx$$

$$\textcircled{2} \int x^3 \cdot \sin x^4 \cdot dx$$

$$\textcircled{3} \int \sin^4 x \cdot \sin^2 x \cdot dx$$

$$\textcircled{4} \int \frac{1}{\sqrt{1-4x^2}} dx$$

$$\textcircled{5} \int \frac{1}{(x-12)^4} dx$$