(1) $\int \frac{\sin x}{\sin(x-a)}\,dx$

$x-a=t$

$x=t+a$

$dx=dt$

$=\int \frac{\sin(t+a)}{\sin t}\,dt$

$=\int \frac{\sin t\cdot\cos a+\cos t\cdot\sin a}{\sin t}\,dt$

$=\int(\cos a+\cot t\cdot\sin a)\,dt$

$=\cos a\int dt+\sin a\int\cot t\,dt$

$=\cos a\cdot t+\sin a\cdot\log\sin t+c$

$=(x-a)\cdot\cos a+\sin a\cdot\log\sin(x-a)+c$

(2) $\int \frac{dx}{\sin(x-a)\cdot\sin(x-b)}$

$=\frac{1}{\sin(a-b)}\int\frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\cdot\sin(x-b)}\,dx$

$=\frac{1}{\sin(a-b)}\int\frac{\sin(x-b)\cdot\cos(x-a)-\cos(x-b)\cdot\sin(x-a)}{\sin(x-a)\cdot\sin(x-b)}\,dx$

$=\frac{1}{\sin(a-b)}\int[\cot(x-a)-\cot(x-b)]\,dx$

$=\frac{1}{\sin(a-b)}(\log\sin(x-a)-\log\sin(x-b))+c$

(3) $\int \frac{\sqrt{\tan x}}{\sin x\cdot\cos x}\,dx$

$=\int\frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}\cdot\cos^2 x}\,dx$

$=\int\frac{\sqrt{\tan x}}{\tan x}\cdot\sec^2 x\,dx$

$\tan x=t$

$\sec^2 x\,dx=dt$

$=\int\frac{\sqrt{t}}{t}\,dt$

$=\int\frac{1}{\sqrt{t}}\,dt=\int t^{-1/2}\,dt$

$=\frac{t^{-1/2+1}}{-1/2+1}+c$

$=\frac{t^{1/2}}{1/2}+c=2\sqrt{t}+c$

$=2\sqrt{\tan x}+c$

(4) $\int \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$

$\sin x-\cos x=t$

$(\cos x+\sin x)\,dx=dt$

$=\int\frac{dt}{\sqrt{1-t^2}}$

$=\sin^{-1}t+c$

$=\sin^{-1}(\sin x-\cos x)+c$

$\sin^2x+\cos^2x-2\sin x\cos x=t^2$

$1-\sin 2x=t^2$

(5) $\int \frac{\sin x}{\sqrt{1+\sin x}}\,dx$

$= -\int \frac{\sin x + 1 - 1}{\sqrt{1+\sin x}}\,dx$

$= \int \frac{1+\sin x}{\sqrt{1+\sin x}}\,dx - \int \frac{1}{\sqrt{1+\sin x}}\,dx$

$= \int \sqrt{1+\sin x}\,dx - \int \frac{1}{\sqrt{1+\sin x}}\,dx$

$= \int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)dx - \int \frac{1}{\sin\frac{x}{2}+\cos\frac{x}{2}}\,dx$

$= \frac{-\cos\frac{x}{2}}{1/2} + \frac{\sin\frac{x}{2}}{1/2} - \int \frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{x}{2} + \frac{1}{\sqrt{2}}\cos\frac{x}{2}\right)}$

$= 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) - \frac{1}{\sqrt{2}}\int \frac{dx}{\sin\frac{\pi}{4}\cdot\sin\frac{x}{2} + \cos\frac{\pi}{4}\cdot\cos\frac{x}{2}}$

$= 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) - \frac{1}{\sqrt{2}}\int \frac{dx}{\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)}$

$= 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) - \frac{1}{\sqrt{2}}\int \sec\left(\frac{x}{2} - \frac{\pi}{4}\right)dx$

$= 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) - \frac{1}{\sqrt{2}}\left[\log\tan\left(\frac{x}{4} - \frac{\pi}{8} + \frac{\pi}{4}\right)\right] + C$

$= 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) - \sqrt{2}\left[\log\tan\left(\frac{x}{4} + \frac{\pi}{8}\right)\right] + C$

Work

(1) $\int \frac{dx}{(3\cos x - 2\sin x)^2}$

(2) $\int \frac{x + \tan^{-1}x^2}{1+x^4}\,dx$

(3) $\int \frac{\cot(\log x)}{x}\,dx$

(4) $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$

(5) $\int \frac{dx}{\cos(x-a)\cdot\cos(x-b)}$