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Department of Civil Engineering

Diploma -4th SEM

04-Lecture Notes on

R.C.C. doubly Beam

Based on limit method

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Doubly Reinforced Beam

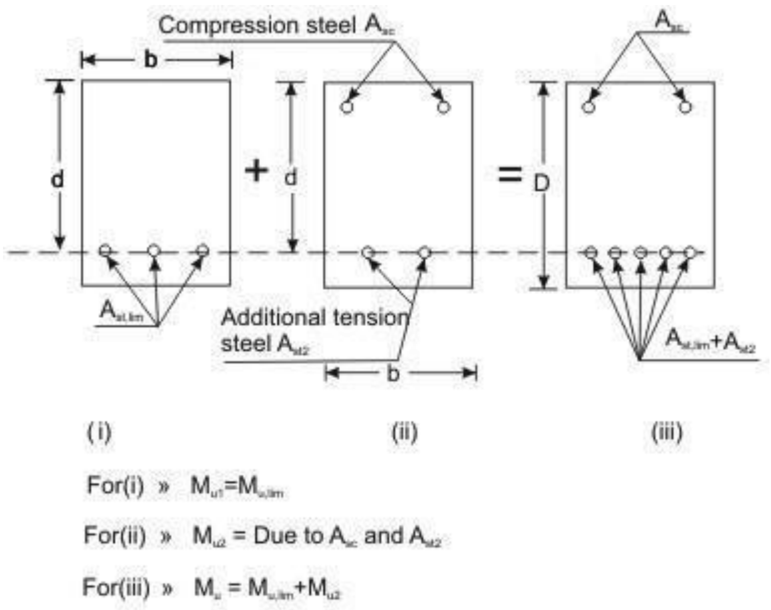

Figure 2.6 Doubly reinforced beam

Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$. Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth (Fig. 2.6). The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st',lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression. Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

(i) Some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.

(ii) The ductility requirement has to be followed.

(iii) The reduction of long term deflection is needed.

Basic Principle

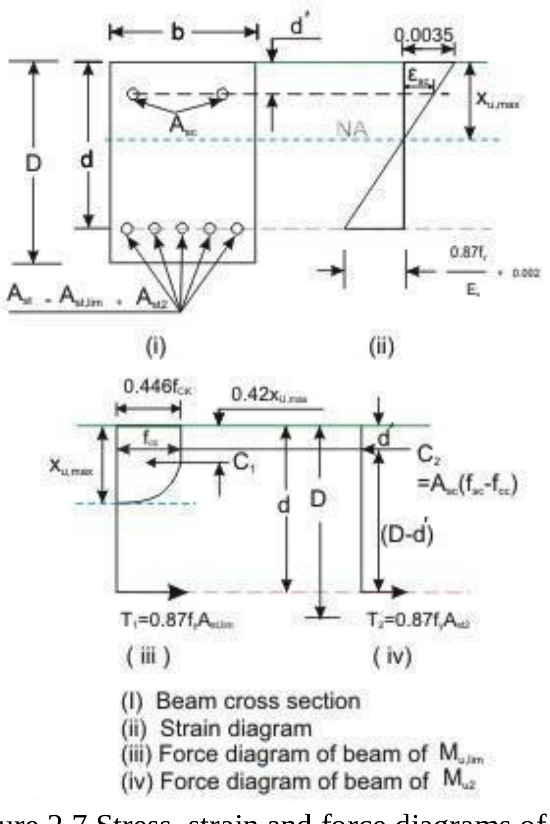

Figure 2.7 Stress, strain and force diagrams of doubly reinforced beam

The moment of resistance M_u of the doubly reinforced beam consists of (i) $M_{u,lim}$ of singly reinforced beam and (ii) M_{u2} because of equal and opposite compression and tension forces (C_2 and T_2) due to additional steel reinforcement on compression and tension faces of the beam (Figs. 2.6 and 7). Thus, the moment of resistance M_u of a doubly reinforced beam is

$M_u = M_{u,lim} + M_{u2}$ (3)

$$M_{u,\lim} = 0.36\frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right) f_{ck} bd^2 \quad (4)$$

Also, $M_{u\,lim}$, can be written

$$M_{u,\lim} = 0.87 A_{st,\lim} f_y (d - 0.416 x_{u,\max}) \quad (5)$$

The additional moment M_{u2} can be expressed in two ways (Fig. 2.7): considering (i) the compressive force C_2 due to compression steel and (ii) the tensile force T_2 due to additional steel on tension face. In both the equations, the lever arm is (d - d'). Thus, we have

$$M_u = A_{sc} (f_{sc} - f_{cc})(d - d') \quad (6)$$

$$M_u = A_{st} (0.87 f_y)(d - d') \quad (7)$$

where A_{sc} = area of compression steel reinforcement

f_{sc} = stress in compression steel reinforcement

f_{cc} = compressive stress in concrete at the level of centroid of compression steel reinforcement

A_{st2} = area of additional steel reinforcement

Since the additional compressive force C_2 is equal to the additional tensile force T_2, we have

$$A_{sc} (f_{sc} - f_{cc}) = A_{st2} (0.87 f_y) \quad (8)$$

Any two of the three equations (Eqs. 6 - 8) can be employed to determine A_{sc} and A_{st2}.

The total tensile reinforcement A_{st} is then obtained from:

$$A_{st} = A_{st1} + A_{st2} \quad (9)$$

$$A_{st1} = p_{t,\lim} \frac{bd}{100} = \frac{M_{u,\lim}}{0.87 f_y (d - 0.42 x_{u,\max})} \quad (10)$$

Determination of f_{sc} and f_{cc}

It is seen that the values of f_{sc} and f_{cc} should be known before calculating A_{sc}. The following procedure may be followed to determine the value of f_{sc} and f_{cc} for the design type of problems (and not for analysing a given section). For the design problem the depth of the neutral axis may be taken as $x_{u,max}$ as shown in Fig. 2.7. From Fig. 2.7, the strain at the level of compression steel reinforcement ε_{sc} may be written as

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{u,\max}}\right) \quad (11)$$

f_{sc} for Cold worked bars Fe 415 and Fe 500

Table 2.1 Values of f_{sc} and ε_{sc}

Stress level	Fe 415		Fe 500	
	Strain ε_{sc}	Stress f_{sc} (N/mm^2)	Strain ε_{sc}	Stress f_{sc} (N/mm^2)
$0.80\ f_{yd}$	0.00144	288.7	0.00174	347.8
$0.85\ f_{yd}$	0.00163	306.7	0.00195	369.6
$0.90\ f_{yd}$	0.00192	324.8	0.00226	391.3
$0.95\ f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975\ f_{yd}$	0.00276	351.8	0.00312	423.9
$1.0\ f_{yd}$	0.00380	360.9	0.00417	434.8

Design type of problems

In the design type of problems, the given data are *b*, *d*, *D*, grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment.

Step 1: To determine $M_{u,lim}$ and $A_{st,lim}$ from Eqs. 4 and 10, respectively.

Step 2: To determine M_{u2}, A_{sc}, A_{st2} and A_{st} from Eqs. 3, 5, 8 and 9, respectively.

Step 3: To select the number and diameter of bars from known values of A_{sc} and A_{st}.

Analysis type of problems

In the analysis type of problems, the data given are *b*, *d*, *d'*, *D*, f_{ck}, f_y, A_{sc} and A_{st}. It is required to determine the moment of resistance M_u of such beams.

Step 1: To check if the beam is under-reinforced or over-reinforced.

First, $x_{u,max}$ is determined assuming it has reached limiting stage using $\frac{x_{u,\max}}{d}$ coefficients as given in cl. 38.1, Note of IS 456. The strain of tensile steel ε_{st} is computed from $\varepsilon_{st} = \frac{\varepsilon_c (d - x_{u,\max})}{x_{u,\max}}$ and is checked if ε_{st} has reached the yield strain of steel:

$$\varepsilon_{st\ at\ yield} = \frac{f_y}{1.15E} + 0.002$$

The beam is under-reinforced or over-reinforced if ε_{st} is less than or more than the yield strain.

Step 2: To determine $M_{u,lim}$ from Eq. 4 and $A_{st,lim}$ from the $p_{t,\ lim}$.

Step 3: To determine A_{st2} and A_{sc} from Eqs. 9 and 8, respectively.

Step 4: To determine M_{u2} and M_u from Eqs. 6 and 3, respectively.

Numerical Problem

Q5-Determine the moment of resistance of an existing beam having the following data: b=350 mm; d=900mm; d' =50mm. Tension reinforcement: 5-20mm HYSD bars (Fe 415); compression reinforcement 2-20 HYSD bars (Fe 415); grade of concrete M15.

Solution

$$A_{st} = 5x\frac{\pi}{4}(20)^2 = 1570.8mm^2;$$

$$A_{sc} = 2x\frac{\pi}{4}(20)^2 = 628.3mm^2$$

$$T = 0.87\ f_y\ A_{st} = 0.87x415x1570.8 = 567120\ N$$

$$C_u = 0.36\ f_{ck}\ x_u b + f_{sc}\ A_{sc} - 0.446\ f_{ck}\ A_{sc}$$

$$=1890x_u + 628.3\ f_{sc} - 4203$$

Let assume x_u =230mm; hence $\frac{3}{7}x_u$ =98.6mm > d'

$$\varepsilon_{sc} = \frac{0.0035(x_u - d')}{x_u} = \frac{0.0035(230 - 50)}{230} = 0.00274$$

Hence from stress-strain curve, we get f_{sc} =351 N/mm^2

C_u = 1890x230 + (628.3x351)-4203=651030 N

This is much more than T_u = 567120 N. Hence take x_u = 190 mm.

$$\varepsilon_{sc} = \frac{0.0035(x_u - d')}{x_u} = \frac{0.0035(190 - 50)}{190} = 0.00258$$

Hence f_{sc} =347 N/mm^2

Cu=572919 N $\simeq$ T

Therefore, M_u = 1890x190x(900-0.416x190) + (628.3x347-4203)x(900-50)

476.5 KN-m