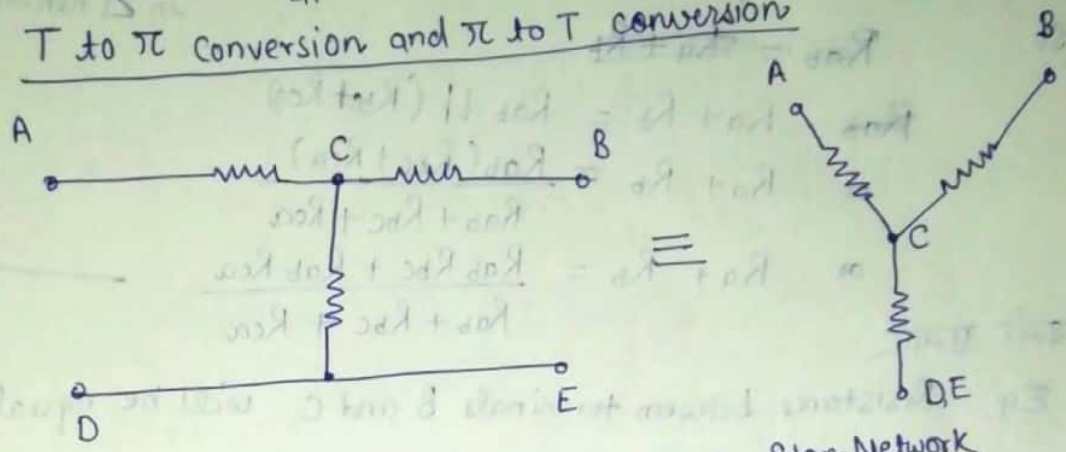
BEE/BMEE

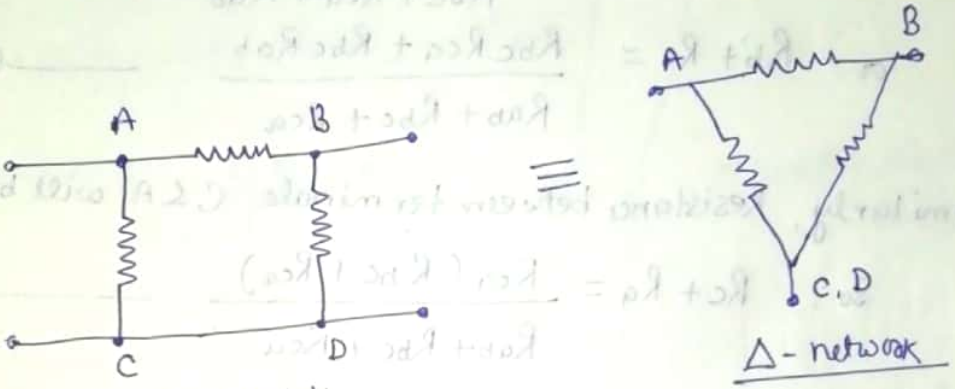
Star-Delta Transformation (स्टार-डेल्टा रूपांतरण)

या

T to π Conversion and π to T conversion

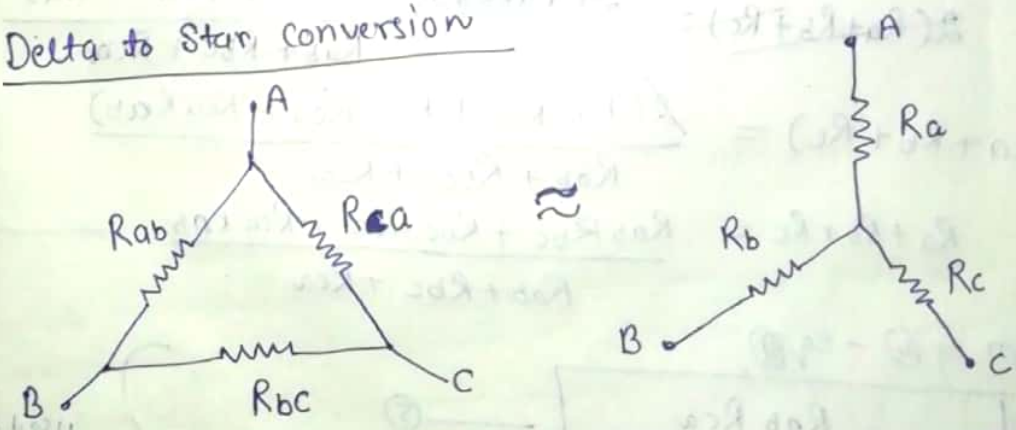


(a) T-Network ≡ (a) Star-Network



π-network ≡ Δ-network

Delta to Star conversion



दो network (star or Delta) तभी equivalent होंगे जबकि किसी दो common terminals जैसे A तथा B, B तथा C तथा C तथा A के बीच equivalent resistance बराबर हों।

For Δ to Y conversion,

Resistance between A & B terminals in star = Resistance between A & B terminals in Δ network.

So. ~~$R_{ab} = R_a + R_b$~~

$R_a + R_b = R_{ab} \,||\, (R_{bc} + R_{ca})$

or $R_a + R_b = \dfrac{R_{ab}(R_{bc} + R_{ca})}{R_{ab} + R_{bc} + R_{ca}}$

or $R_a + R_b = \dfrac{R_{ab}R_{bc} + R_{ab}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$ ——— (1)

~~इसी प्रकार~~

Eq. Resistance between terminals B and C will be equal

So $R_b + R_c = \dfrac{R_{bc}(R_{ca} + R_{ab})}{R_{bc} + R_{ca} + R_{ab}}$

or $R_b + R_c = \dfrac{R_{bc}R_{ca} + R_{bc}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$ ——— (2)

Similarly, Resistance between terminals C & A will be equal.

So, $R_c + R_a = \dfrac{R_{ca}(R_{bc} + R_{ca})}{R_{ab} + R_{bc} + R_{ca}}$ ——— (3)

Adding eq 1, 2, and 3, we get

~~$2(R_a + R_b + R_c) = \dfrac{2(R_{ab} + R_{bc} + R_{ca})}{R_{ab} + R_{bc} + R_{ca}}$~~

$2(R_a + R_b + R_c) = \dfrac{2(R_{ab}R_{bc} + R_{bc}R_{ca} + R_{ca}R_{ab})}{R_{ab} + R_{bc} + R_{ca}}$

$R_a + R_b + R_c = \dfrac{R_{ab}R_{bc} + R_{bc}R_{ca} + R_{ca}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$ ——— (4)

from eq (4) − eq (2),

$$\boxed{R_a = \frac{R_{ab}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}}$$ ——— (5)

from eq (4) − eq (3),

$$\boxed{R_b = \frac{R_{bc}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}}$$ ——— (6)

Similarly eq (4) − eq (1),

$$\boxed{R_c = \frac{R_{ca}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}}$$ ——— (7)

used for Δ to Y Conversion

~~For~~ Y to Δ conversion

from eq ⑤, ⑥ and ⑦,

$$R_a R_b + R_b R_c + R_c R_a = \frac{R_{ab}^2 R_{bc} R_{ca}}{(R_{ab}+R_{bc}+R_{ca})^2} + \frac{R_{ab} R_{bc}^2 R_{ca}}{(R_{ab}+R_{bc}+R_{ca})^2} + \frac{R_{ab} R_{bc} R_{ca}^2}{(R_{ab}+R_{bc}+R_{ca})^2}$$

$$\text{or } R_a R_b + R_b R_c + R_c R_a = \frac{R_{ab}^2 R_{bc} R_{ca} + R_{ab} R_{bc}^2 R_{ca} + R_{ab} R_{bc} R_{ca}^2}{(R_{ab}+R_{bc}+R_{ca})^2}$$

$$\text{or } R_a R_b + R_b R_c + R_c R_a = \frac{R_{ab} R_{bc} R_{ca} \cancel{(R_{ab}+R_{bc}+R_{ca})}}{(R_{ab}+R_{bc}+R_{ca})^{\cancel{2}}}$$

$$\text{or } R_a R_b + R_b R_c + R_c R_a = \frac{R_{ab} R_{bc} R_{ca}}{R_{ab}+R_{bc} R_{ca}} \quad \text{——— ⑧}$$

from eq ⑧ ÷ eq ⑦

$$\frac{R_a R_b + R_b R_c + R_c R_a}{R_c} = R_{ab}$$

$$\text{or } R_{ab} = \frac{R_a R_b}{R_c} + R_a + R_b$$

Similarly from, eq ⑧ ÷ eq ⑤,

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a}$$

and

$$R_{ca} = R_c + R_a + \frac{R_c R_a}{R_b}$$

Used for Y to Δ conversion