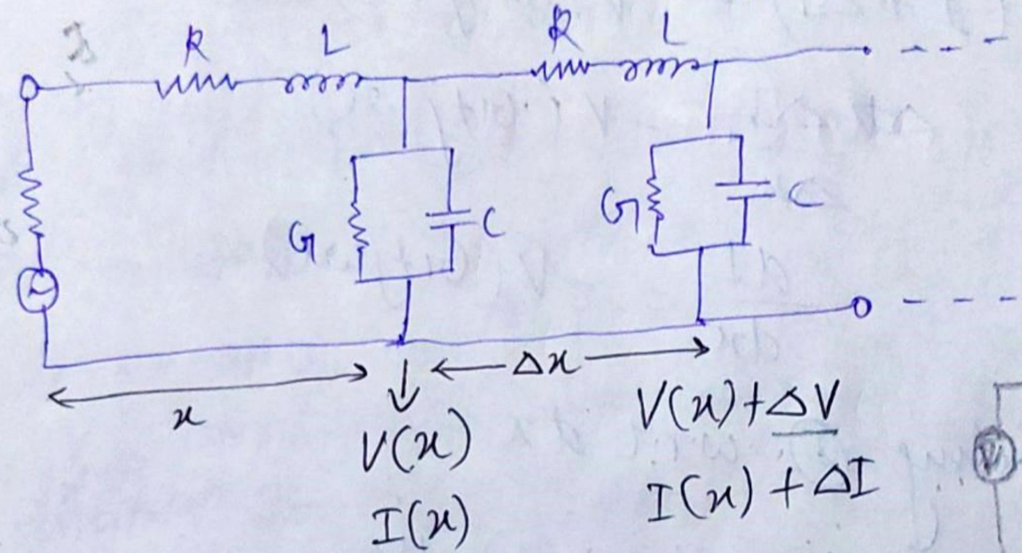
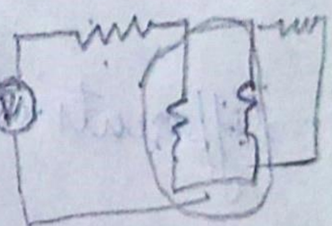
Transmission Lines Equations

→ Let the length of the transmission line be 'l', and the primary constants be R, L, G, and C.

→ A small section of the line 'XY' is considered, of length 'Δx' ~~and at~~ at a distance of 'x' from source.





Considering a small section Δx

series impedance $= (R+j\omega L)\Delta x$

shunt ~~impedance~~ admittance $= (G+j\omega C)\Delta x$

→ Voltage drop occurs due to elements in series arm of the ~~voltage~~ circuit.

→ Current drop occurs due to elements in shunt arm of the circuit

So drop in voltage, will be

$$V,I = f(x)$$

$$V - (V + \Delta V) = I(R + j\omega L)\Delta x$$

$$-\Delta V = I(R + j\omega L)\Delta x$$

$$= \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -(R + j\omega L)I \quad \text{— (1)}$$

and similarly drop in current

$$I - (I + \Delta I) = V(G + j\omega C)\Delta x$$

$$\frac{\Delta I}{\Delta x} = -V(G + j\omega C)$$

$$\frac{dI}{dx} = -V(G + j\omega C) \quad \text{— (2)}$$

Differentiating (2) w.r.t dx

$$\frac{d^2 I}{dx^2} = -(G + j\omega C)\frac{dV}{dx}$$

putting (1) in above equation

$$\frac{d^2 I}{dx^2} = (G + j\omega C)(R + j\omega L)I$$

Let $\gamma^2 = (G + j\omega C)(R + j\omega L)$, then

$$\boxed{\frac{d^2 I}{dx^2} = \gamma^2 I}$$

Differentiating ① and putting ① into it, will result into

$$\boxed{\frac{d^2V}{dx^2} = \gamma^2 V}$$

On simplifying $\frac{d^2V}{dx^2} - \gamma^2 V = 0$ is a general form of differential equation and its solution can be obtained as:

$$V = A \cdot e^{\gamma x} + B \cdot e^{-\gamma x}$$

$$e^{\gamma x} = e^{\gamma x} = \cosh(\gamma x) + \sinh(\gamma x)$$

$$e^{-\gamma x} = \cosh(\gamma x) - \sinh(\gamma x)$$

$$V = A(\cosh(\gamma x) + \sinh(\gamma x)) + B(\cosh(\gamma x) - \sinh(\gamma x))$$

$$V = (A+B)\cosh(\gamma x) + (A-B)\sinh(\gamma x)$$

$$V = a\cosh(\gamma x) + b\sinh(\gamma x) \quad -③$$

where $a = A+B$, $b = A-B$

$$\frac{dV}{dx} = a \cdot \gamma \sinh(\gamma x) + b \cdot \gamma \cdot \cosh(\gamma x)$$

~~Following similar~~ $I = \frac{-1}{(R+j\omega L)} \frac{dV}{dx}$

$$I = \frac{-1}{(R+j\omega L)} \cdot \left[a \cdot \gamma \cdot \sinh(\gamma x) + b \cdot \gamma \cosh(\gamma x)\right]$$

on $\gamma^2 = (R+j\omega L)(G+j\omega C)$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega L)}$$

$$I = -\sqrt{\frac{G+j\omega C}{R+j\omega L}} \left[a\sinh(\gamma x) + b\cosh(\gamma x)\right]$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$I = -\frac{1}{Z_0}\left[a \sinh(\gamma x) + b \cosh(\gamma x)\right] \quad -(4)$$

at $x = 0$, $I = I_s$, $V = V_s$. $I_s = \frac{-b}{Z_0}$

from (5), at $x = 0$, $V = V_s$.

$\Rightarrow \quad a = V_s$

from (4) at $x = 0$, $I = I_s$.

$$I = \frac{-b}{Z_0} \qquad b = -I_s Z_0$$

then $V = V_s \cdot \cosh(\gamma x) - I_s Z_0 \sinh(\gamma x)$

$$I = I_s \cdot \cosh(\gamma x) - \frac{V_s}{Z_0} \sinh(\gamma x)$$

at a ~~dis~~ distance of 'x' from the source

then

at $x = l$.

$$V = V_L \cdot \cosh(\gamma l) - I_L \cdot Z_0 \sinh(\gamma l)$$

$$I = I_L \cosh(\gamma l) - \frac{V_L}{Z_0} \sinh(\gamma l)$$