Question:- There are 5 pulleys in a system of Pulleys and single rope is passed over all pulleys. Then find the effort required to lift 2000 N load. If $\eta = 60\%$ then find effort required.

Solve:- Given data - Second System of pulleys

$W = 2000\ N$, $n = 5$

To find ① ~~M.A.~~ $P = ?$

$$\left[M.A. = \frac{W}{P} = n = VR \right]$$

$$M.A. = \left[\frac{W}{P} = n \right]$$

$$5 = n = \frac{W}{P}$$

$$\frac{W}{P} = 5$$

$$\frac{2000}{P} = 5$$

$$P = \frac{2000}{5}$$

$$[P = 400 N]$$

Case - II

$\eta = 60\% = 0.6$

To find out :- $P = ?$

$$\eta = \frac{M.A.}{V.R.}$$

$$M.A. = \eta \times V.R.$$

$$\frac{W}{P} = \eta \times V.R.$$

$$\frac{2000}{P} = 0.6 \times (5)$$

$$P = \frac{2000}{5 \times 0.6}$$

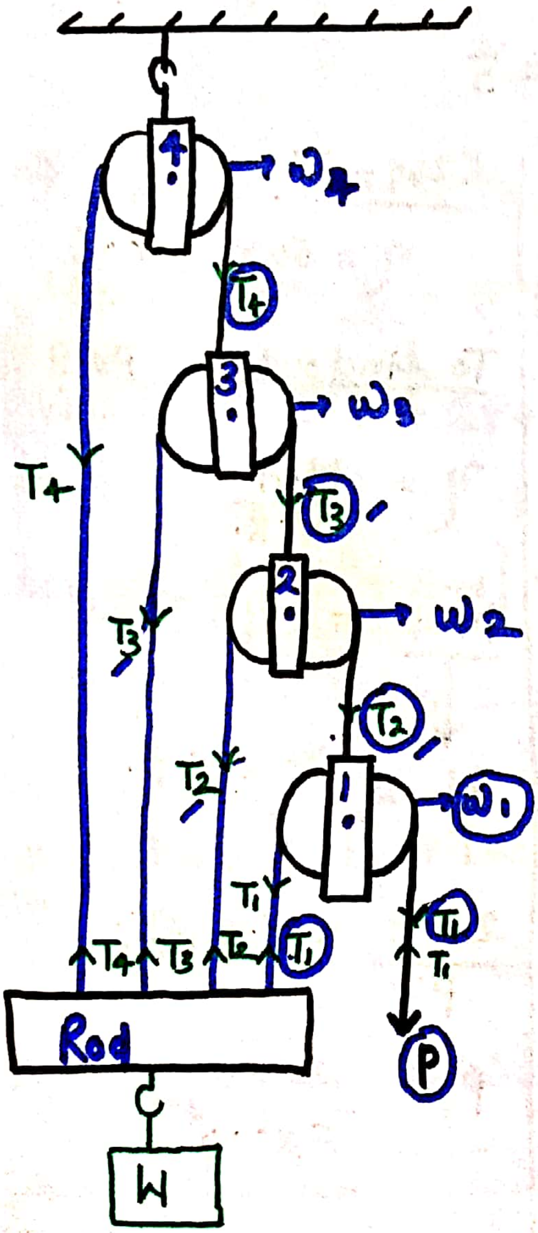
$$\left[P = \frac{2000}{3} = 666.6667\ N \right]$$

3- Third System of pulleys :-

→ Inverse of the First pulleys system.

→ One fixed and many moving pulleys.

→ One end of every rope is attached with one horizontal rod.



At equilibrium-

$T_1 = P$

$T_2 = T_1 + T_1 = 2T_1 = 2P$

$T_3 = 2T_2 = 2 \times 2P = 2^2 P$

$T_4 = 2T_3 = 2 \times 2^2 P = 2^3 P$

Similarly if there are 'n' pulleys-

$$\boxed{T_n = 2^{n-1} P}$$

Again, Weight $W = T_1 + T_2 + T_3 + T_4 + \dots + T_n$

$$W = P + 2P + 2^2P + 2^3P + \dots 2^{n-1}P$$

$$W = P(1 + 2 + 2^2 + 2^3 + \dots 2^{n-1})$$

it is in G.P., which sum are-

$$\Rightarrow \frac{a(r^n - 1)}{r - 1}$$

here $a = 1$, $r = 2$, $n = n$

$$\boxed{W = P(2^n - 1)}$$

$$\therefore \Rightarrow \boxed{M.A. = \frac{W}{P} = (2^n - 1)}$$

if $\eta = 100\% = 1$ (ideal)

$$\boxed{M.A. = V.R. = (2^n - 1)}$$

$$\because \left\{ \eta = \frac{M.A.}{V.R.} \right\}$$

Case - II :- If weight of the pulleys are $w_1, w_2, w_3 \ldots w_n$ then -

$$T_1 = P$$

$$T_2 = 2T_1 + w_1 = \underline{2P + w_1}$$

$$T_3 = 2T_2 + w_2 = 2\times(2P + w_1) + w_2 = \underline{2^2P + 2w_1 + w_2}$$

$$T_4 = 2T_3 + w_3 = 2[2^2P + 2w_1 + w_2] + w_3$$

$$T_4 = \underline{2^3P + 2^2w_1 + 2w_2 + w_3}$$

$$\Rightarrow \quad W = T_1 + T_2 + T_3 + T_4$$

$$W = P + (2P + w_1) + (2^2P + 2w_1 + w_2) + (2^3P + 2^2w_1 + 2w_2 + w_3)$$

$$W = (2^3P + 2^2P + 2P + P) + (2^2w_1 + 2w_1 + w_1) + (2w_2 + w_2) + w_3$$

$$W = P\underbrace{[2^3 + 2^2 + 2 + 1]}_{G.P. \to (2^n - 1)} + w_1\underbrace{[2^2 + 2 + 1]}_{G.P.} + w_2\underbrace{[2 + 1]}_{G.P.} + w_3$$

n = No. of pulleys.

$W \Rightarrow$

$$W = P[2^n - 1] + w_1[2^{n-1} - 1] + w_2[2^{n-2} - 1] + w_3$$

if $w_1 = w_2 = w_3 = w$

$$W = P[2^n - 1] + \{w[2^{n-1} - 1] + w[2^{n-2} - 1] + \cdots w_n(2^1 - 1)\}$$

$$\cancel{W = P(2^n - 1) + w[(2^{n-1} - 1) + (2^{n-2} - 1) + 1]}$$

$$W = \underline{P(2^n - 1)} + \underline{w}\left[\underline{(2^{n-1} - 1) + (2^{n-2} - 1) + (2^1 - 1)}\right]$$

$$\boxed{W = \underline{P(2^n - 1)} + \underline{w}[\underline{2^n - n - 1}]}$$

$$\left[M.A. = \frac{W}{P} = \frac{P(2^n - 1) + w(2^n - n - 1)}{P}\right]$$

For ideal machine -

if $\eta = 100\%$. $\eta = \frac{M.A.}{V.R.}$

$$\boxed{M.A. = V.R. = \frac{P(2^n - 1) + W(2^n - n - 1)}{P}}$$

Again if we put $W = 0$

$$M.A. = V.R. = \frac{P(2^n - 1) + 0(2^n - n - 1)}{P}$$

$$M.A. = V.R. = \frac{\cancel{P}(2^n - 1)}{\cancel{P}}$$

$$\boxed{M.A. = V.R. = 2^n - 1}$$

Case - I //