Now: to find Direction of Resultant:-

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{(OA + AD)}$$

$$\tan\alpha = \frac{(Q\sin\theta)}{(P + Q\cos\theta)}$$

$$\left\{\alpha = \tan^{-1}\left[\frac{(Q\sin\theta)}{P + Q\cos\theta}\right]\right\}$$ direction of Resultant.

Question:- 2-forces of 15 N and 12 N are at an angle of 60° to each-other. Find the resultant force. Both the forces are pulling in Nature.

Given data:- $P = 15N$, $Q = 12N$, $\alpha = 60°$ ⇓ $\theta = 60°$

To find :- Resultant (R).

Solve :-

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{15^2 + 12^2 + 2\times15\times12\cos60°}$$

$$R = \sqrt{225 + 144 + 180}$$

$$\left\{R = \sqrt{549} = 23.43\,N\,(\text{pull})\right\}_{\text{Magnitude}}$$

Direction :-

$$\left\{\tan\theta = \frac{Q\sin\theta}{P + Q\cos\theta}\right\}$$

$$\tan\theta = \frac{12\times\sin60°}{15 + 12\times\cos60°}$$

$$\tan\theta = \frac{10.392}{21} = 0.4948$$

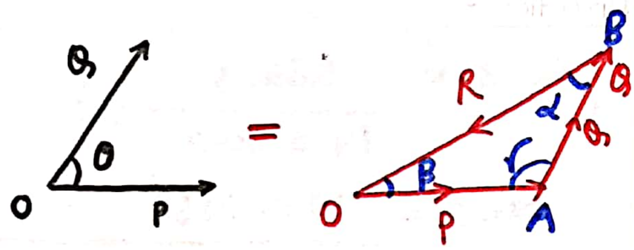
$$\theta = \tan^{-1}(0.4948)$$

$\boxed{\theta = 26°20'}$ from 15 Newton.

LAWS OF FORCES:-

(1) TRIANGLE LAW OF FORCES:-

" If two forces are acting as the adjacent sides of a triangle taken in order then the closing side of the triangle represents the resultant in opposite direction."



$P \propto \sin\alpha \Rightarrow P = K\sin\alpha \Rightarrow K = \frac{P}{\sin\alpha}$

$Q \propto \sin\beta \Rightarrow K = \frac{Q}{\sin\beta}$

$R \propto \sin\gamma \Rightarrow K = \frac{R}{\sin\gamma}$

$$\left[\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}\right]$$ from Sine Rule.

NOTE:-

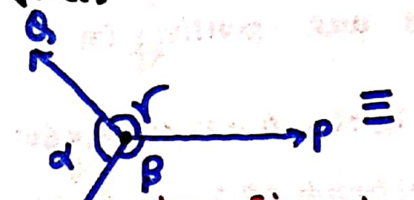
$\vec{OA} + \vec{AB} + \vec{BO} = 0$

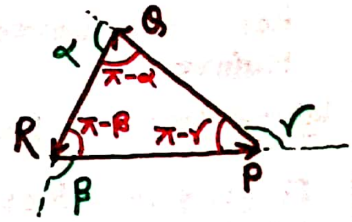
$\vec{OA} + \vec{AB} = -\vec{BO}$

$\{\vec{OA} + \vec{AB} = \vec{OB}\}$

(2) LAMI'S THEOREM:-

If three forces are acting at a point and they are in equilibrium then every force is directly proportional to sine of angle between the other 2-forces.





from Sine rule-

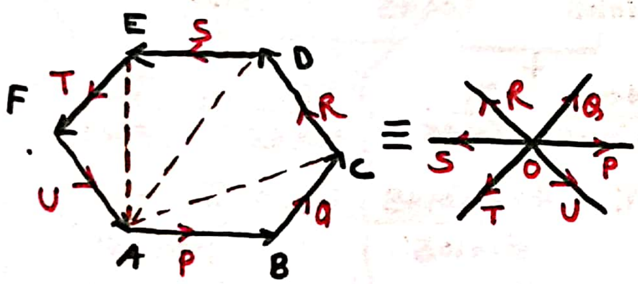
R

$$\frac{P}{\sin(\pi-\alpha)} = \frac{Q}{\sin(\pi-\beta)} = \frac{R}{\sin(\pi-\gamma)}$$

$$\therefore \left[\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}\right]$$

LAW OF POLYGON OF FORCES :-

If many forces are acting at a point and if it can be represented by sides of polygon in series by magnitude and direction. then these forces will be in equilibrium.



ABCDEF is a polygon.

$$\vec{AB} + \vec{BC} = \vec{AC} \quad \text{—— (i)}$$

$$\vec{AC} + \vec{CD} = \vec{AD} \quad \text{—— (ii)}$$

$$\vec{AD} + \vec{DE} = \vec{AE} \quad \text{—— (iii)}$$

$$\vec{AE} + \vec{EF} = \vec{AF} \quad \text{—— (iv)}$$

from equation (i), (ii), (iii) & (iv)

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} = \vec{AF}$$

$$\therefore \left[\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} + \vec{FA} = 0\right]$$

are in equilibrium.